Team 351
Problem A

A Thicker Martian Atmosphere

Abstract

In this paper, we analyze Mars' atmosphere if it had 0.2 bar of pressure from vaporizing the polar caps. Such atmosphere would be stripped over time due to several factors, in which would be discussed in this paper is solar winds. Energy from the solar wind is absorbed into the particles in the atmosphere, giving them the required energy to escape Mars' gravity. From this principle, we develop a model that strips Mars' exosphere from solar wind energy, which resulted in an H_2O loss of $1.6 \times 10^{26}\,\mathrm{s}^{-1}$ or $4.8\,\mathrm{kg\,s}^{-1}$. Using the ideal gas law, with constant volume of the atmosphere from surface to the start of the exosphere and temperature to sustain water vapor, we made a mathematical model for the decrease of pressure over time.

1 Introduction

Mars is the fourth planet in the solar system, nicknamed the “red planet” after its seemingly red surface. Mars has a surface similar to that of Earth, with craters, volcanoes, valleys, deserts, and ice caps made of mostly frozen water and a thin layer of frozen carbon dioxide (dry ice). It has an average distance of 1.52 au from the sun, a radius of 3389.5 km, a mass of 6.4×10^{23} kg, a surface gravity of $3.72\,\mathrm{m\,s^{-2}}$, and an effective temperature of 210 K [1, 2, 3].

The southern and northern polar cap have volumes of $1.6 \times 10^{6}\,\mathrm{km^3}$ and $1.6 \times 10^{6}\,\mathrm{km^3}$, respectively [4, 5]. Assuming that they primarily consists of frozen water, the total mass of the water is 2.9×10^{21} kg. Comparing it to the total mass of the present-day Mars' atmosphere, which is 2.5×10^{16} kg, the vaporized water takes about 99.99 % of the atmosphere. We can safely assume that the atmosphere comprises only water vapor (H_2O).

Ancient Mars had a thicker atmosphere —about as thick as Earth's— than today's Mars, making the former more conducive to harboring life. This difference is due to many factors, most notably solar wind stripping, considering Mars' weak magnetosphere [2]. Solar wind is an electrically charged particle emitted by the sun in all directions, moving at high speeds. It is known as high-temperature plasma, conductive, with a supersonic speed of $400\,\mathrm{km\,s^{-1}}$ at a distance of 1 au [2, 6]. In Mars, escape happens in the exosphere, 230 km above the surface [3].

The mean solar wind energy flux at 1 au with scaling of R^2_{AU} is $1.5 \times 10^{-4}\,\mathrm{W\,m^{-2}}$. At distance 1.52 au (Mars' orbit), the flux is $7.4 \times 10^{-4}\,\mathrm{W\,m^{-2}}$ [1, 7].

To simplify our problem, let us state several assumptions.

1. As previously stated, due to the abundance of vaporized water from vaporized ice caps filling 99.99 % of the atmosphere, we can safely assume that the atmosphere on Mars consists only of H_2O.

2. Unlike Earth, Mars does not generate its own magnetic field. Small patches of magnetic field from magnetized crust exist, but they are negligible, so it is safe to assume that Mars has no magnetic field protecting it from solar winds [8].

3. Temperature on Mars, according to data from Rover Environmental Monitoring Station (REMS), has repeating pattern that stays the same each year. With the assumption that the sun doesn't change throughout the years, the average temperature on mars each year can be assumed to be constant at 210 K [9]. Due to Mars' low temperature, escape by thermal velocity can also be neglected.

4. Solar activities is assumed to be constant, including solar wind. Therefore, the solar wind energy flux is constant. In addition, we only assume that solar wind is the only playing factor in Mars' atmosphere loss.

2 Notations Used

Symbols	Meaning	Numeric Value
r_{Mars}	Mars radius	3389.5 km
m_{Mars}	Mars mass	6.4×10^{23} kg
V_{atm}	Volume of Mars' atmosphere from surface to the start of the exosphere	
P	Average pressure	
n	Number of particles in moles	
T_{Mars}	Mars' effective temperature	210 K
v_{escape}	Velocity required to escape Mars	
M_{H_2O}	Molar mass of H_2O	$18\,\mathrm{g\,mol^{-1}}$
G	Gravitational constant	$6.7 \times 10^{-11}\,\mathrm{m\,kg^{-3}\,s^{-2}}$
R	Universal gas constant	$8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$

3 Derivation of the Model

Solar energy flux is used to determine how much H_2O is stripped out of the atmosphere every second by knowing how much energy H_2O needs to escape Mars' gravity. Assuming that all of the energy that is absorbed goes to kinetic energy to achieve escape velocity, the total amount of H_2O that could potentially escape can be calculated.

Given that only half of Mars' surface is being hit by solar wind each time, the total energy that is absorbed by Mars around the start of the exosphere can be calculated from the flux and half of the surface area of the start of the exosphere region. The exosphere starts at the height of 230 km above the surface and with Mars' radius of 3389.5 km, half of the exosphere's surface area is $8.23 \times 10^7\,\mathrm{km}^2$ [1].

The velocity required to escape Mars' gravity is given by the following equation [2]:

$$v_{\mathrm{escape}} = \sqrt{\frac{2Gm_{\mathrm{Mars}}}{r_{\mathrm{Mars}}}}. \tag{1}$$

Using the data from [1], the escape velocity for Mars equates to $1.6 \times 10^5\,\mathrm{m\,s^{-1}}$. This means that the kinetic energy required for a single H_2O 3.8×10^{-16} J.

The amount of energy absorbed by half of Mars' exosphere is 60.9 GW. With that much power, the amount of H_2O escaping Mars equals to $1.6 \times 10^{26}\,\mathrm{s^{-1}}$ which is close to the amount escaping mars from [10]. In terms of mass, that is $4.8\,\mathrm{kg\,s^{-1}}$ of H_2O.

The existence of water vapor in the atmosphere of Mars requires temperature that could sustain the vapor phase. Temperature for water vapor in Mars where the pressure is 0.2 bar can be calculated using the Clausius-Clapeyron Equation and enthalpy of vaporization of $40.657\,\mathrm{kJ\,mol^{-1}}$ for pure water in 1 atm [11, 12]. The resulting temperature is 332.04 K or 58.89^oC.

Knowing the amount of H_2O escaping Mars, the change in pressure over time can be described with an equation derived from the ideal gas law [13]

$$P(t) = n_{\mathrm{H}_2O}(t)\frac{RT_{\mathrm{mars}}}{V_{\mathrm{atm}}},$$

namely,

$$\Delta P = m\dot{_{\mathrm{H}_2O}}\frac{RT_{\mathrm{mars}}}{V_{\mathrm{atm}}M_{\mathrm{H}_2O}}t. \tag{2}$$

meaning, the amount of pressure decreased over time is

$$P(t) = P_0 - m_{\dot{\mathrm{H}}_2O} \frac{RT_{\mathrm{mars}}}{V_{\mathrm{atm}} M_{\mathrm{H}_2O}} t. \quad (3)$$

A graph is created from equation (3) to describe the decrease of pressure over time.

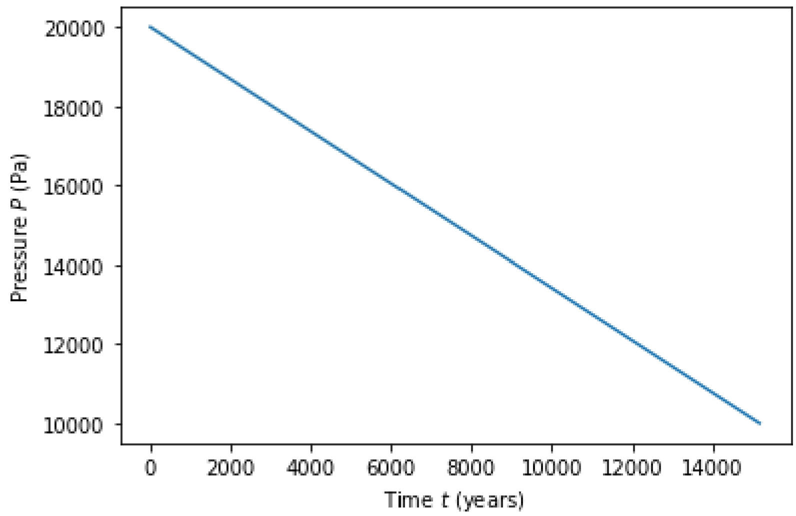

Figure 1: Change in pressure as a function of time in years.

The time required to lower the pressure from 0.2 bar to 0.1 bar is 15 180 years from equation (3). It changes linearly through time due to its constant volume and temperature.

4 Conclusions and Possible Improvements

We found that the rate of loss of H_2O is $4.8\,\mathrm{kg\,s^{-1}}$ and a time of 15 180 years is required to lower the pressure from 0.2 bar to 0.1 bar.

The model used in this paper assumes that the sun throw out a consistent solar wind and solar wind energy flux when actually it's inconsistent. Also, the volume of Mars' atmosphere is assumed to be constant, even though particles are stripped from the atmosphere, lowering the atmosphere's volume. Furthermore, Mars' temperature is not constant due to the changing pressure and greenhouse effect from water vapor. The model itself is not bad, as it predicted the loss of atmosphere quite accurately from [10]. A more accurate model can be made by considering the solar wind energy flux inconsistencies, volume change due to the stripping of the atmosphere, and temperature changes from pressure changes and greenhouse effect.

5 References

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6 Appendix

Here we append the Python code used to produce Figure 1.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy import optimize

R = 8.314
T = 332.04
V = 35509592341.534546
mr_h2o = 18
mass_loss_rate_h2o = 4.8365822796383435

def delta_p(t):
  return (mass_loss_rate_h2o*t)*R*T/(V*mr_h2o)

reduced = 0.1e5 # pascal

def delta_p_mod(t):
  return (mass_loss_rate_h2o*t)*R*T/(V*mr_h2o) - reduced

def pressure_over_time(t):
  return 0.2e5 - delta_p(t)

sol = optimize.root_scalar(delta_p_mod, x0=1e2, x1=1e3, method="secant")
print(sol)

t = np.linspace(0, sol.root, 100)
pressures = pressure_over_time(t)

plt.plot(t, pressures)
plt.xlabel("Time $t$ (s)")
plt.ylabel("Pressure $P$ (Pa)")
plt.show()
```