Team 262

Problem B

Quadcopter Stability in Wind

Abstract

In this paper we analyze the forces in a quadcopter to find the maximum wind speed which allows it to stay hovering stationarily on a target location. For the quadcopter to remain stationary, the force of the wind it resists must be balanced by the quadcopter's opposing thrust. Using this principle, we develop a mathematical model for a quadcopter with a cube-shaped body with dimensions of 30 cm × 30 cm × 30 cm and weight 1.5 kg, for which the maximum resistible wind speed is 17.712 $m \cdot s^{-1}$.

1. Introduction

A quadcopter is a popular form of UAV (Unmanned Aerial Vehicle). The thrust from the rotors plays a key role in maneuvering and keeping the copter airborne. Its small size and swift maneuverability enables the user to perform flying routines that include complex aerial maneuvers. But for conducting such maneuvers, precise angle handling of the copter is required (Khan, 2014, 1).

In this paper we construct a mathematical model to estimate the maximum velocity the wind surrounding a quadcopter can attain without resulting in a significant deviation of the quadcopter from its target location. More specifically, consider a quadcopter with a mass of 1.5 kg and four rotor blades, each generating up to 7 Newtons of thrust. We shall determine the maximum wind speed which allows the quadcopter to stay within 20 cm of its target location.

To simplify the problem, we make the following assumptions.

1. The quadcopter's body is cube-shaped with dimensions of 30 cm × 30 cm × 30 cm and four tubular arms emanating diagonally from the midpoints of its four vertical edges (see Figure 1). Since the body contributes most of the drag, we assume that the drag from the arms are negligible.

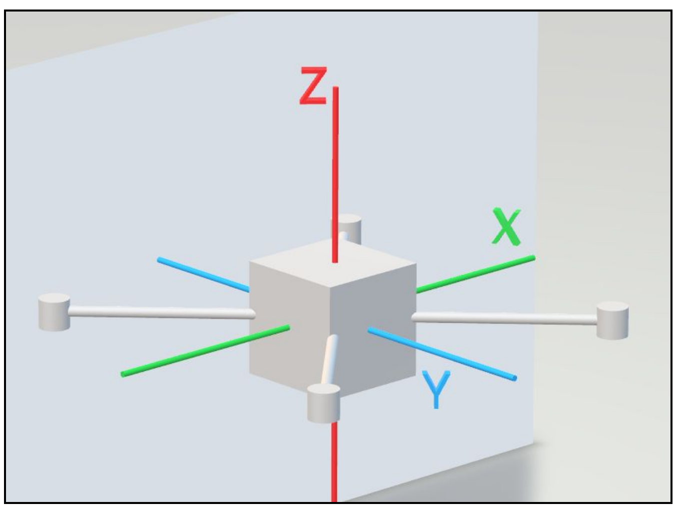

Figure 1. The shape of the quadcopter without the rotor blades.

2. The wind flows only in the direction of X and Y axes and is perpendicular to one of these axes (never diagonally), and hence so is the movement of the quadcopter itself. The fact that the quadcopter has 90° rotational symmetry about the Z-axis allows us to perform the analysis in two dimensions by viewing the quadcopter from, e.g., the positive Y-axis (see Figure 2).

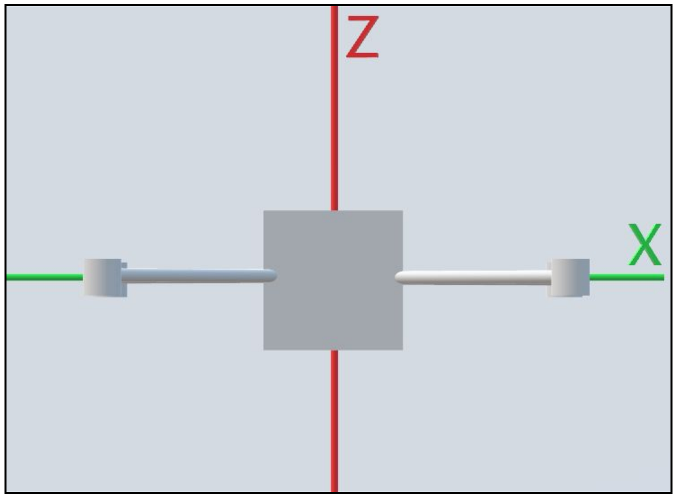

Figure 2. The quadcopter as viewed from the positive Y-axis.

3. The quadcopter is already hovering in a stationary position above the target location; we do not consider its earlier flight maneuvers.

This paper is organized as follows. In Section 2, we explain some of the notations used throughout this paper. In Section 3, we derive our model by considering, firstly, the simpler case where the wind is neglected, and secondly, the main case where wind is taken into account. The conclusion is stated in Section 4, namely that the quadcopter with the above specifications is able to resist wind speed up to 17.712 $m \cdot s^{-1}$.

2. Notation Used

In the following table we list some symbols which will be used in our model. More symbols are to be defined in the subsequent sections.

Symbols	Meaning	Numeric Value
F_t	Force of thrust	
F_g	Force of quadcopter's weight	14.715 N
P_w	Pressure of wind	
F_w	Force of wind	
v_w	Velocity of wind	

3. Derivation of the Model

Let θ be the deviation angle of the quadcopter, i.e., the angle between the positive Z-axis and the normal of the quadcopter (see Figure 3).

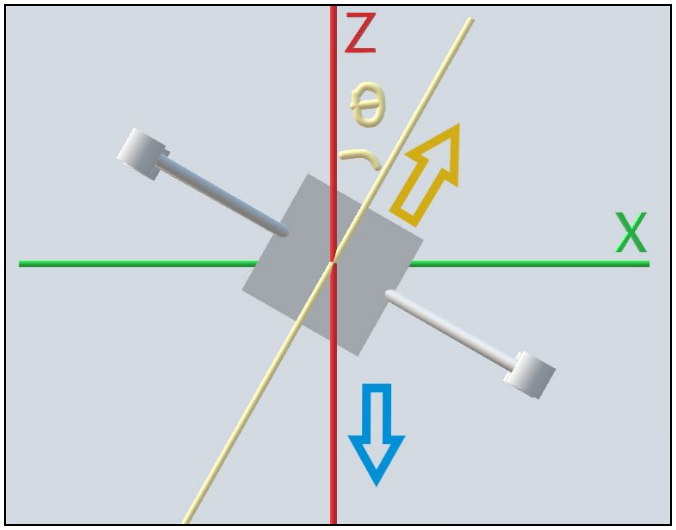

Figure 3. The deviation angle θ of the quadcopter, and the normal of the quadcopter, i.e., the yellow line. The orange and blue arrows represent the directions of the thrust and weight force, respectively.

We shall derive a mathematical model which expresses v_w as a function of θ. This will be achieved in two stages: we first look into the simpler case where wind disturbances are neglected (Section 3.1) before proceeding to the main case where these are taken into account (Section 3.2). All graphs below are generated using Python; the codes are available in the Appendix.

3.1 Neglecting the Wind

If the wind is neglected, for the drone to remain stationary, its thrust projected to the Z-axis must be equal to the weight force, i.e.,

$$F_t \cos(\theta) = F_g .$$

This gives

$$\theta = \arccos\left(\frac{F_g}{F_t}\right). \tag{1}$$

Here we have expressed the angle θ for which the quadcopter remains stationary for any given values of F_t and F_g.

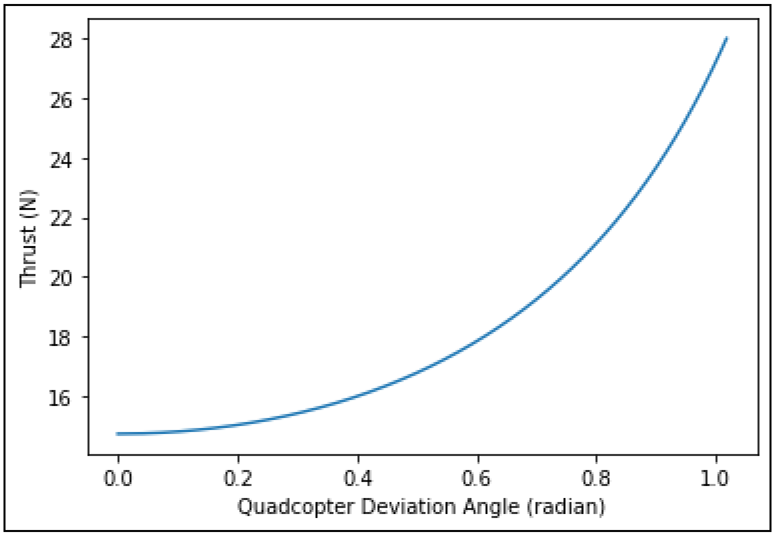

Figure 4. Plot of F_t versus θ satisfying the relationship (1) for F_g = 14.175 N, i.e., the amount of thrust the quadcopter exerts to hover stationarily as a function of the deviation angle.

When the thrust is at its maximum of 28 Newtons, the maximum deviation of its angle is around 1 radian. All four rotors are pushing with the same thrust. In this situation the quadcopter should have a deviation angle of zero radian, but it does not (see Figure 4). Hence, there exists a force in the X-axis that is preventing the quadcopter from staying upright; this is equal to the projection of the thrust of the quadcopter to the X-axis, i.e., $F_t \sin\theta$.

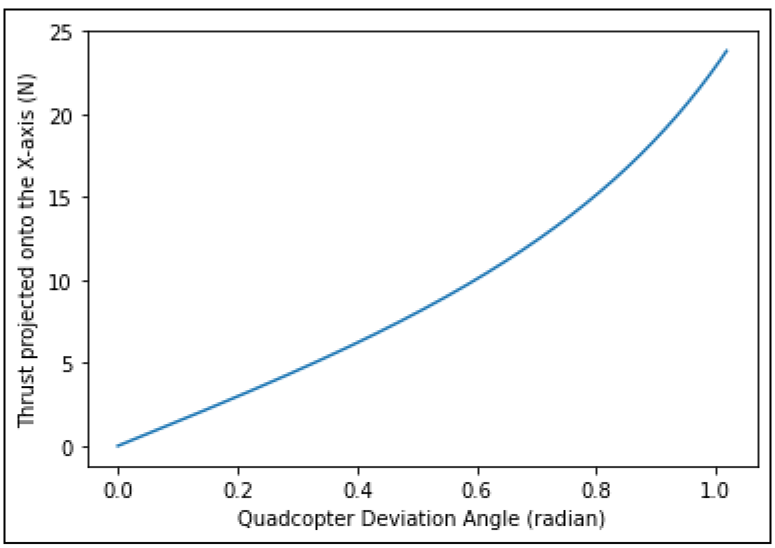

Figure 5. Plot of $F_t \sin\theta$ versus θ satisfying the relationship (1) for F_g = 14.175 N, i.e., the amount of thrust projected onto the X-axis the quadcopter exerts to hover stationarily as a function of the deviation angle.

3.2 Incorporating the Wind

When the wind is taken into account, the surface area that is hit by the wind must be considered. The total surface area that is facing the wind can be calculated using the equation

$$A(\theta) = \cos(\theta)A_1 + \sin(\theta)A_2 ,$$

where A_1 and A_2 are the areas of the surfaces facing the wind. In the case of $A_1 = A_2 = A$, we have

$$A(\theta) = A[\cos(\theta) + \sin(\theta)] . \tag{2}$$

Using the definition of pressure,

$$P_w = \frac{F_w}{A(\theta)} .$$

By a result in (Richardson, n.d.), the pressure of the wind can be calculated as a function of the wind speed, i.e.,

$$P_w = 0.613\ v_w^2 .$$

Combining the last two equations, one obtains an expression for the maximum wind speed as a function of the deviation angle θ, namely,

$$v_w = \sqrt{\frac{F_w}{0.613A(\theta)}} . \tag{3}$$

The force exerted in the X-axis by the quadcopter is equal to the maximum wind force it could resist. That is,

$$F_w = F_t \sin\theta = \frac{F_g \sin\theta}{\cos\theta} .$$

Substituting this and (2) into (3) gives

$$v_w = \sqrt{\frac{F_g \sin\theta}{0.613\cdot\cos\theta\cdot A[\cos(\theta)+\sin(\theta)]}} .$$

For $F_g = 14.715$ N and $A = 0.3^2$ m^2 = 0.09 m^2, we have the expression

$$v_w = \sqrt{\frac{266.721 \sin\theta}{\cos\theta\cdot[\cos(\theta)+\sin(\theta)]}}$$

for the wind speed as a function of the deviation angle; this is graphed in Figure 6.

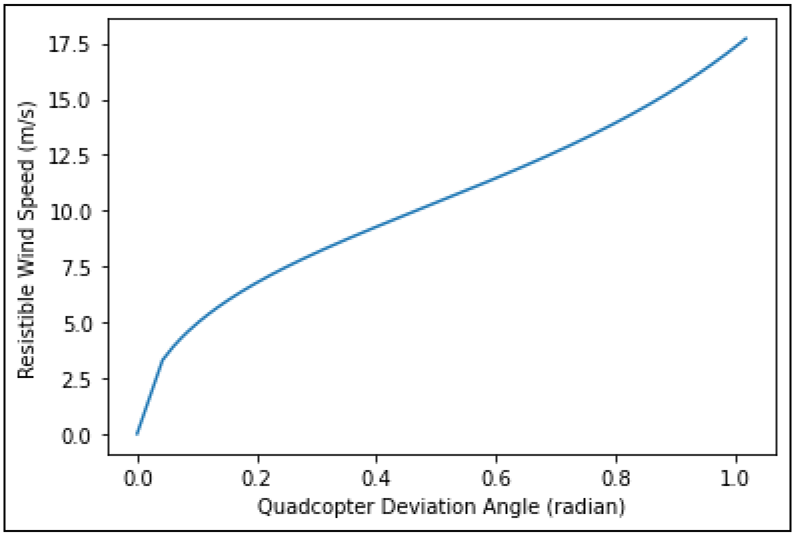

Figure 6. The wind speed the quadcopter could resist as a function of its deviation angle.

Hence, the theoretical maximum wind speed the quadcopter could resist (the value of v_W for $\theta = 1.018$ radians, the deviation angle corresponding to the maximum thrust force) is 17.712 $m{\cdot}s^{-1}$.

For a comparison, if the lengths of the sides of the quadcopter's body is changed to 40 cm (the mass remains the same), its surface area increases: $A = 0.4^2\ m^2 = 0.16\ m^2$, and we have

$$v_W = \sqrt{\frac{150.031 \sin\theta}{\cos\theta \cdot [\cos(\theta)+\sin(\theta)]}}$$

which is graphed in Figure 7. Notice that the maximum resistible wind speed is lower.

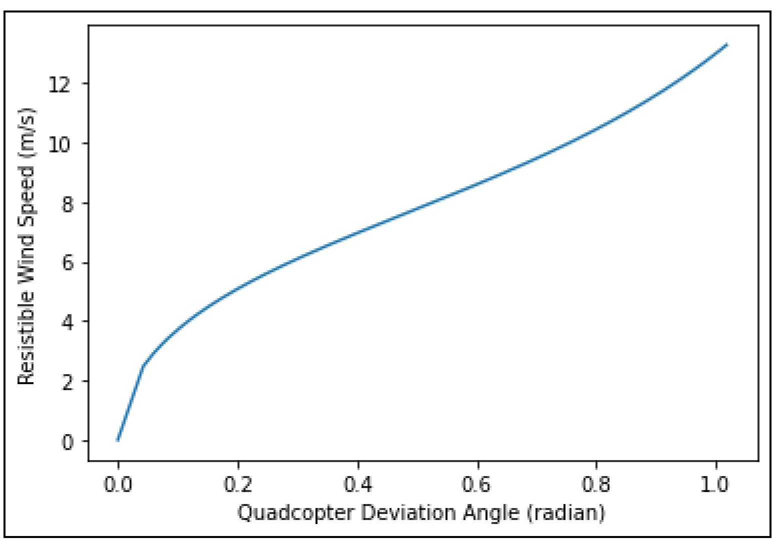

Figure 7. The wind speed the quadcopter with body dimensions of 40 cm × 40 cm × 40 cm could resist as a function of its deviation angle.

4. Conclusion and Possible Improvements

We found that the quadcopter with body dimensions of 30 cm × 30 cm × 30 cm and a mass of 1.5 kg can resist wind speed up to 17.712 $m \cdot s^{-1}$.

The method used in this paper neglects the diagonal movement of the quadcopter since, from the pilot's point of view, moving along the X or Y axis is more preferable. This creates a setting in which the maximum resistible wind speed can be calculated more easily, although it does not account for winds with varying directions and turbulences. Improvements could be made by taking into account winds from all directions, thereby advancing the model's accuracy.

5. References

Khan, M. (2014, August). Quadcopter Flight Dynamics. *International Journal of Scientific & Technology Research*, *3*(8), 1. Retrieved November 8th, 2020, from http://www.ijstr.org/final-print/aug2014/Quadcopter-Flight-Dynamics.pdf.

Richardson. (n.d.). *Minimum Design Loads for Buildings and Other Structures*. Retrieved November 8th, 2020, from http://richardson.eng.ua.edu/Former_Courses/DWRS_fa11/Notes/ASCE_7_05_Chapter_6.pdf.

6. Appendix

Here we append the Python code used to produce Figures 4-7.

```
import numpy as np
import matplotlib.pyplot as plt

Ft = np.linspace(14.715, 28, 1000)
theta = np.arccos(14.715/Ft)

# Plotting theta and thrust
plt.plot(theta, Ft)
plt.xlabel('Quadcopter Deviation Angle (radian)')
plt.ylabel('Thrust (N)')
plt.show()

# Calculating total surface area
def surface_area(theta, a1, a2):
  a = np.cos(theta) * a1 + np.sin(theta) * a2
  return a

# Plotting theta and thrust
Fx = Ft * np.sin(theta)
plt.plot(theta, Fx)
plt.xlabel('Quadcopter Deviation Angle (radian)')
plt.ylabel('Thrust in X (N)')
plt.show()

# Plotting theta and resistible wind speed
v = np.sqrt(Fx/(0.613*surface_area(theta, 0.3*0.3, 0.3*0.3)))
plt.plot(theta, v)
plt.xlabel('Quadcopter Deviation Angle (radian)')
plt.ylabel('Resistible Wind Speed (m/s)')
plt.show()

# Plotting theta and thrust
v = np.sqrt(fx/(0.613*surface_area(thet, 0.4*0.4, 0.4*0.4)))
plt.plot(thet, v)
plt.xlabel('Quadcopter Deviation Angle (radian)')
plt.ylabel('Resistible Wind Speed (m/s)')
plt.show()
```